CHAPTER 23—WHERE YOU'RE GOING

Along with understanding how the homework problems work, you should also:

1.) Talk intelligently about conductors and non-conductors (insulators).

2.) From the examination of a charge configuration, be able to tell in what direction the force on a test charge placed somewhere in proximity would be.

3.) Understand the parameters used in Coulomb's Law.

4.) Understand what electric fields are.

5.) Be able to interpret electric field lines. Be able to construct electric field lines.

6.) Given an electric field, be able to determine the electric force that acts on a point charge in the field.

7.) Be able to execute a Newton's Second Law problem with electric fields involved.

8.) Knowing the electric field function for a point charge, understand how to derive an expression for the electric field vector (both magnitude and direction) generated by a charge configuration, where the charge distribution can either be discrete or continuous. That is, understand the technique required to derive electric field functions for the following situations:

NOTE: Students rightly assume that everything they are presented with is accurate and error free, primarily because most of their books they use have been written by committees with loads of resources and lots of time to spend on proof reading the material (though even then you occasionally find an error in early versions of books like Serway's). Obviously, no teacher wants to provide his or her students with information that is incorrect.

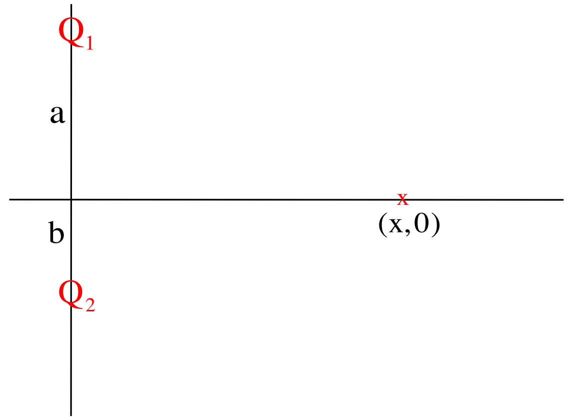
Having said that, you should know that it takes a long, long, long time to generate the graphics involved in a Power Point like the one you are about to examine, not to mention the long, long time it takes to use the Formula Editor to write out the math. There are, in all probability, way over a thousand individual steps involved in the making of this thirty page-plus Power Point. IF YOU FIND SOMETHING THAT DOESN'T MAKE SENSE IN THE PAGES THAT FOLLOW, there is a possibility that I've simply made a dumb error. (Just my proof-reading this--not generating it, but simply re-reading what I've created--is a good hour's worth of work).

I've proof read this monster twice, but there's still a chance I might have messed something up somewhere along the line. If you find a goof, don't be put off (mistakes happen). Just let me know and if I agree with your assessment, I'll make corrections.

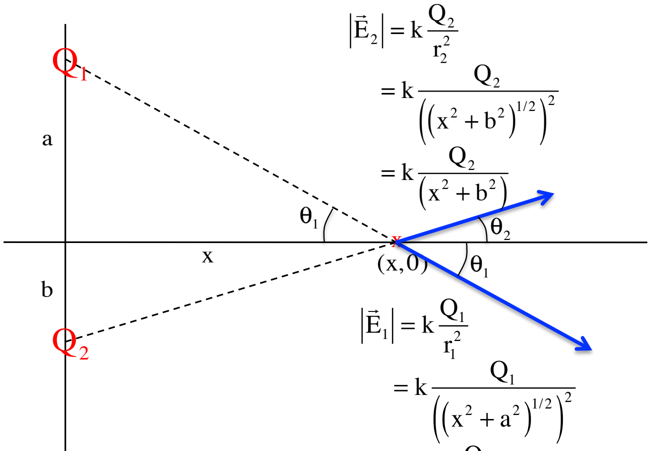
A.) For the point charge configuration shown, derive the expression for the electric field as it exists at (x,0).

This is a relatively simple problem that is covered on pgs 718-719 in your text. I spent some time writing it out before realizing it was there, though, so I'm including it below.

The general procedure for doing these problems is to separately determine the DIRECTION of the *electric field* generated by each charge, then attach to each directional arrow the field's magnitude. The field lines will probably have to broken into x and y-directions, so you'll probably have to use the sine and cosine trick shown in class. If you can exploit symmetry, do so. All of this is shown for this problem on the next three pages.



Shown is the direction and magnitude for each charge-produced electric field:



$$\left|\vec{E}_2\right| = k\frac{Q_2}{r_2^2} = k\frac{Q_2}{\left(\left(x^2+b^2\right)^{1/2}\right)^2} = k\frac{Q_2}{\left(x^2+b^2\right)}$$

$$\left|\vec{E}_1\right| = k\frac{Q_1}{r_1^2} = k\frac{Q_1}{\left(\left(x^2+a^2\right)^{1/2}\right)^2} = k\frac{Q_1}{\left(x^2+a^2\right)}$$

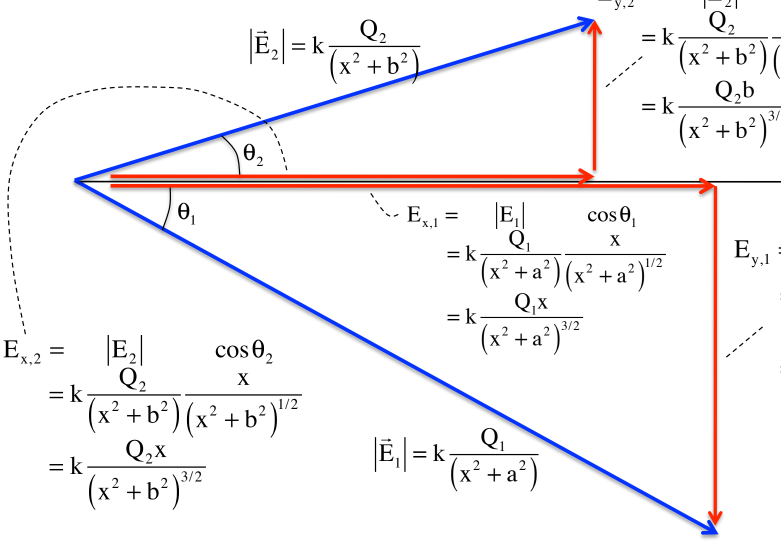
Note that the trig functions associated with the angles can be were written as shown:

$$\cos\theta_1 = \frac{x}{\left(x^2+a^2\right)^{1/2}} \qquad \sin\theta_1 = \frac{a}{\left(x^2+a^2\right)^{1/2}}$$

$$\cos\theta_2 = \frac{x}{\left(x^2+b^2\right)^{1/2}} \qquad \sin\theta_2 = \frac{b}{\left(x^2+b^2\right)^{1/2}}$$

The x and y component of E_1 and E_2:

$$\cos\theta_1 = \frac{x}{\left(x^2+a^2\right)^{1/2}}$$
$$\sin\theta_1 = \frac{a}{\left(x^2+a^2\right)^{1/2}}$$
$$\cos\theta_2 = \frac{x}{\left(x^2+b^2\right)^{1/2}}$$
$$\sin\theta_2 = \frac{b}{\left(x^2+b^2\right)^{1/2}}$$



$$\left|\vec{E}_2\right| = k\frac{Q_2}{\left(x^2+b^2\right)}$$

$$\left|\vec{E}_1\right| = k\frac{Q_1}{\left(x^2+a^2\right)}$$

$$E_{y,2} = \left|E_2\right| \sin\theta_2 = k\frac{Q_2}{\left(x^2+b^2\right)}\frac{b}{\left(x^2+b^2\right)^{1/2}} = k\frac{Q_2 b}{\left(x^2+b^2\right)^{3/2}}$$

$$E_{x,1} = \left|E_1\right| \cos\theta_1 = k\frac{Q_1}{\left(x^2+a^2\right)}\frac{x}{\left(x^2+a^2\right)^{1/2}} = k\frac{Q_1 x}{\left(x^2+a^2\right)^{3/2}}$$

$$E_{y,1} = \left|E_1\right| \sin\theta_1 = k\frac{Q_1}{\left(x^2+a^2\right)}\frac{a}{\left(x^2+a^2\right)^{1/2}} = k\frac{Q_1 a}{\left(x^2+a^2\right)^{3/2}}$$

$$E_{x,2} = \left|E_2\right| \cos\theta_2 = k\frac{Q_2}{\left(x^2+b^2\right)}\frac{x}{\left(x^2+b^2\right)^{1/2}} = k\frac{Q_2 x}{\left(x^2+b^2\right)^{3/2}}$$

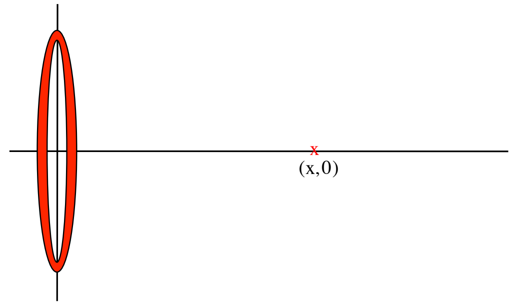
4.)

Putting it all together, taking into consideration the signs as defined by the coordinate system created for the problem, we get:

$$\vec{E} = \left(\left|E_2\right|\cos\theta_2 + \left|E_1\right|\cos\theta_1 \right)\hat{i} + \left(\left|E_2\right|\sin\theta_2 - \left|E_1\right|\sin\theta_1 \right)\hat{j}$$
$$= \left(k\frac{Q_2 x}{\left(x^2+b^2\right)^{3/2}} + k\frac{Q_1 x}{\left(x^2+a^2\right)^{3/2}} \right)\hat{i} + \left(k\frac{Q_2 b}{\left(x^2+b^2\right)^{3/2}} - k\frac{Q_1 a}{\left(x^2+a^2\right)^{3/2}} \right)\hat{j}$$

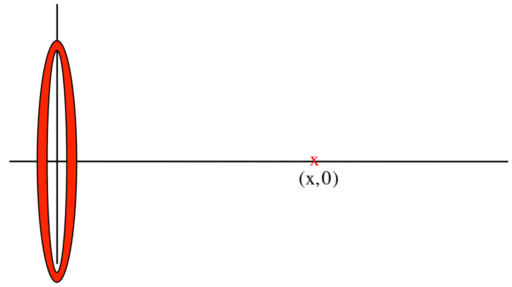
5.)

B.) A ring situated in the y-z plane (as shown) has -Q's worth of charge on it. Derive an expression for the electric field as it exists at (x,0).



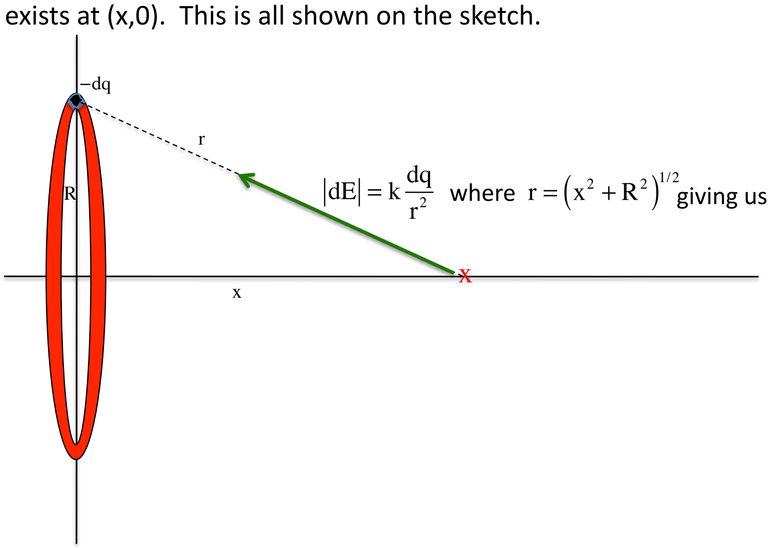
See pg 722.

C.) A ring situated in the y-z plane (as shown) has a constant $-\lambda$'s worth of charge-per-unit-length on it. Derive an expression for the electric field as it exists at (x,0).



Although this is similar to the problem on pg 722, the linear charge density function $-\lambda$ gives it a little different twist. How to deal with that is shown on the next several pages.

We start by defining a differential charge dq residing on a tiny bit of hoop of length ds. Being a point charge, we know its electric field function will be of the form $k\,{}^{q}\!/_{r^2}$, where the charge "q" is really "dq." We can use that expression and our knowledge of how to determine the direction of an electric field (green arrow—remember, it's the direction a positive test charge would accelerate if put in the field at the point of interest) to characterize both the direction and magnitude of the differential electric field dE as it exists at (x,0). This is all shown on the sketch.

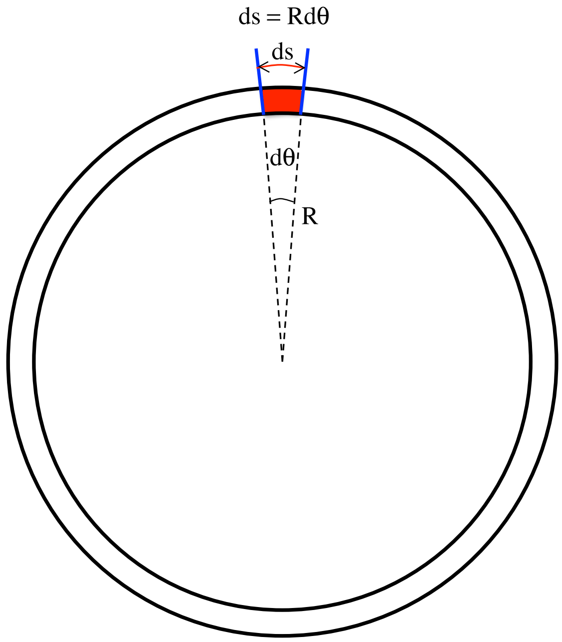


$|dE| = k\frac{dq}{r^2}$ where $r = \left(x^2 + R^2\right)^{1/2}$ giving us $|dE| = k\frac{dq}{(r)^2}$

$$= k\frac{dq}{\left(\left(x^2 + R^2\right)^{1/2}\right)^2}$$

$$= k\frac{dq}{\left(x^2 + R^2\right)}$$

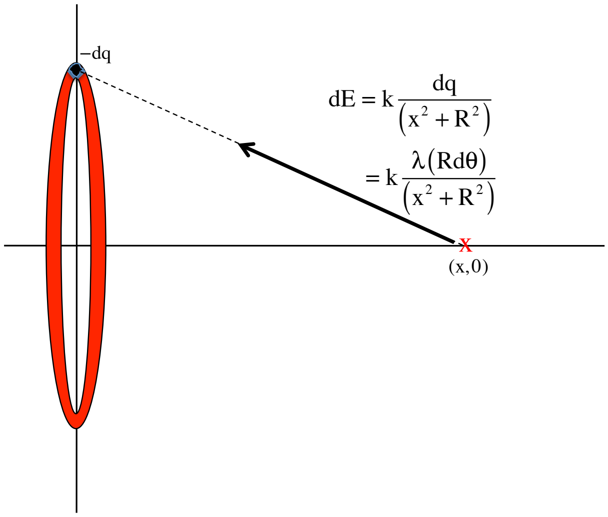
There is a relationship between the length-of-hoop ds upon which dq resides, the angle subtended by that section ($d\theta$) and the radius of the arc R. It is the consequence of the definition of the radian and is such that $ds = Rd\theta$ (this assumes that $d\theta$ is measured in radians--see sketch).



Note that knowing the differential length ds (i.e., $Rd\theta$), we can multiple that length by the charge-per-unit-length λ to determine the differential charge dq residing on ds. That is,

$$\begin{aligned} dq &= \lambda(ds) \\ &= \lambda(Rd\theta) \end{aligned}$$

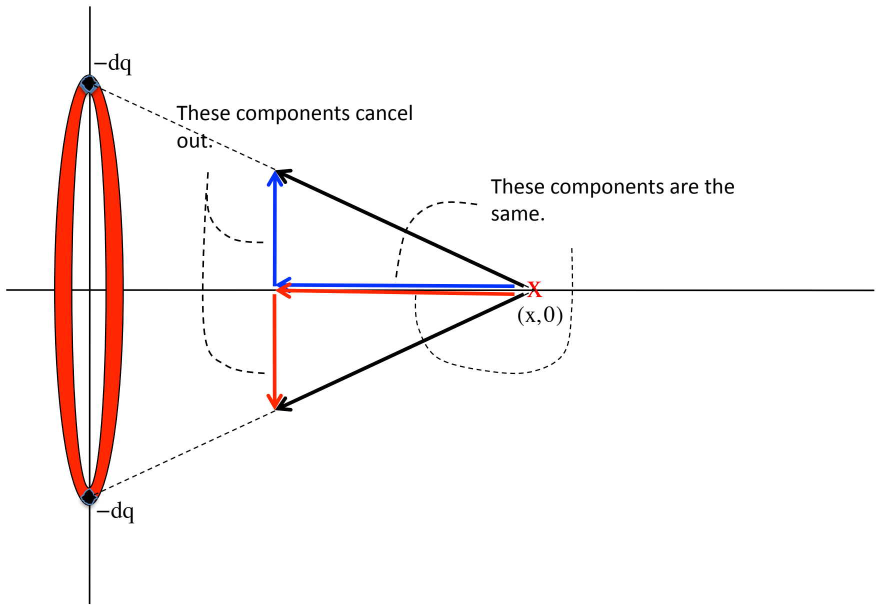
With all of this information, we can now write the differential electric field dE in terms of the charge-per-unit-length λ, the radius R of the arc and the differential angle $d\theta$ that subtends ds. That is:



$$\begin{aligned} dE &= k\frac{dq}{(x^2 + R^2)} \\ &= k\frac{\lambda(Rd\theta)}{(x^2 + R^2)} \end{aligned}$$

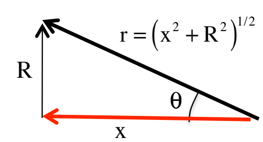
Note that the linear-charge-density function is treated as though it was positive even though it was defined as negative. What's important is to determine the MAGNITUDE of dE due to the charge on ds using the MAGNITUDE of the charge-per-unit-length function. The DIRECTION of the field is determined using the standard approach, which is to say asking the question, "In what direction would a positive test charge accelerate (due to dq) if placed at the point of interest . . . (x,0) in this case . . . and released." That direction is the direction of the electric field at that point.

Exploiting symmetry: Notice that there is another differential amount of charge "–dq" on the opposite side of the hoop. Notice also the direction that that differential electric field sets up. Due to the symmetry of the charge distribution, you can see that the vertical component of each of the two fields cancels one another out leaving only the horizontal components with which to deal. Schematically, this looks like:



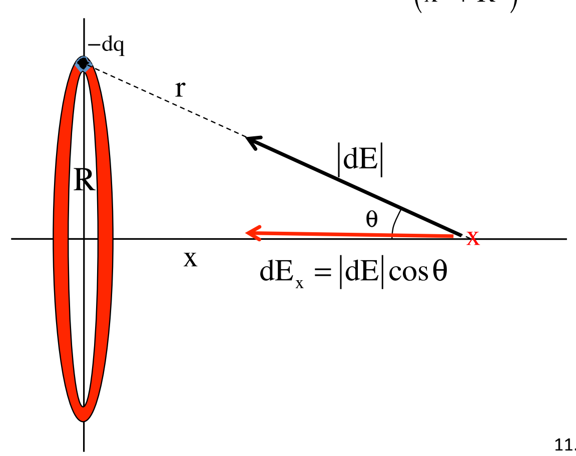
Apparently, given the symmetry, all we need to determine is the x-component of dE (that will be $dE_x = |dE|\cos\theta$), then use Calculus to sum that component around the entire hoop. That will give us the net x-component of the electric field due to all the dq's.

Being clever, we note that the cosine (i.e., the ratio of the side adjacent to θ to the hypotenuse) is just "x/r" (see sketch). Incorporating that information into our x-component expression, we get:



$$\cos\theta = \frac{x}{\left(x^2+R^2\right)^{1/2}}$$

$$\begin{aligned} dE_x &= \quad dE \qquad\qquad \cos\theta \\ &= \left(k\frac{\lambda R\,d\theta}{\left(x^2+R^2\right)}\right)\left(\frac{x}{\left(x^2+R^2\right)^{1/2}}\right) \\ &= \left(k\frac{\lambda R x}{\left(x^2+R^2\right)^{3/2}}\,d\theta\right) \end{aligned}$$



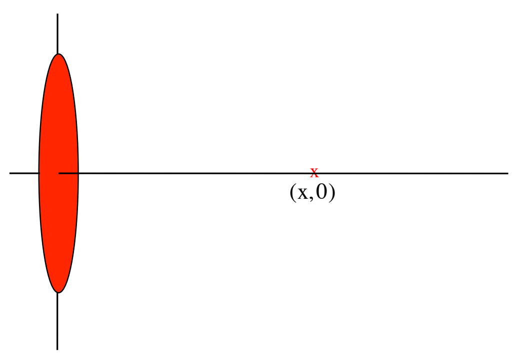
Noting that the integral is over $d\theta$, and observing that none of the variable in the expression are functions of θ, we can write:

$$
\begin{aligned}
\left|\vec{E}\right| &= \int dE\cos\theta \\
&= \int_{\theta=0}^{2\pi}\left(k\frac{\lambda Rx}{\left(x^2+R^2\right)^{3/2}}d\theta\right) \\
&= \left(k\frac{\lambda Rx}{\left(x^2+R^2\right)^{3/2}}\right)\int_{\theta=0}^{2\pi}d\theta \\
&= \left(k\frac{R\lambda x}{\left(x^2+R^2\right)^{3/2}}\right)(2\pi-0)
\end{aligned}
$$

As the total charge on the hoop is $Q=(2\pi R)\lambda$, we can rearrange this expression to eliminate λ and get the same solution as was acquired in the book's hoop problem. Putting all of that into vector form, we get:

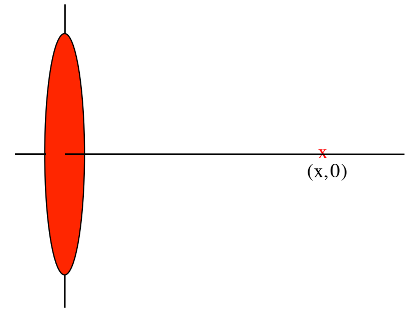
$$
\begin{aligned}
\vec{E} &= \left(k\frac{R\lambda x}{\left(x^2+R^2\right)^{3/2}}\right)(2\pi)\hat{i} \\
&= \left(k\frac{\left[(2\pi R)\lambda\right]x}{\left(x^2+R^2\right)^{3/2}}\right)\hat{i} \\
&= \left(k\frac{Q\ x}{\left(x^2+R^2\right)^{3/2}}\right)\hat{i}
\end{aligned}
$$

D.) A solid disk situated in the y-z plane (as shown) has Q's worth of charge on it. Derive an expression for the electric field as it exists at (x,0).



Although this problem is done on pg 722 in your text, I will lay it out anyhow (it is a problem that will be connected to a lab down the line).

Some preliminaries:



First, notice is that if there is Q's worth of charge total distributed uniformly over the surface area of the disk (where that area is πR^2), the constant, surface-charge-density (the amount of charge per unit area) is:

$$\sigma = \frac{Q_{disk}}{\pi R^2}$$

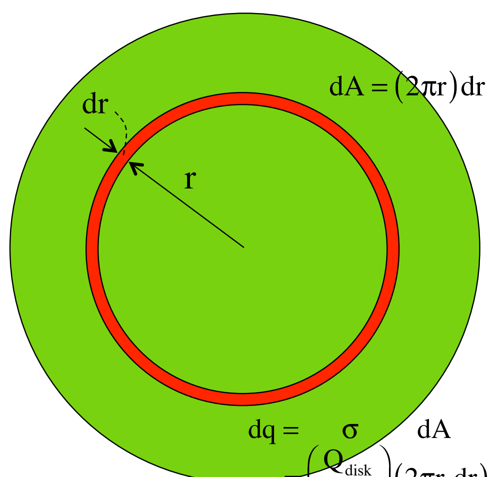
σ can also be written in differential form. That is, ratio of the differential charge dq to the differential area dA upon which dq resides will also yield σ. Mathematically, this is:

$$\sigma = \frac{dq}{dA} \quad \Rightarrow \quad dq = \sigma dA$$

If we combine the two relationships, we get an expression for dq in terms of Q, R, r and dr. That is:

$$\begin{aligned} dq &= \quad \sigma \qquad dA \\ &= \left(\frac{Q_{disk}}{\pi R^2}\right)(2\pi r\ dr) \\ &= \left(\frac{2Q_{disk}}{R^2}\right)(r\ dr) \end{aligned}$$

Looking at the plate head-on, all the variable we discussed and their equivalents are presented. The red area is a thin swath of charge dq whose radius is r, whose differential thickness is dr and whose differential area is dA. The differential area dA is mathematically equal to the product of the swath circumference and its differential thickness dr. That is:



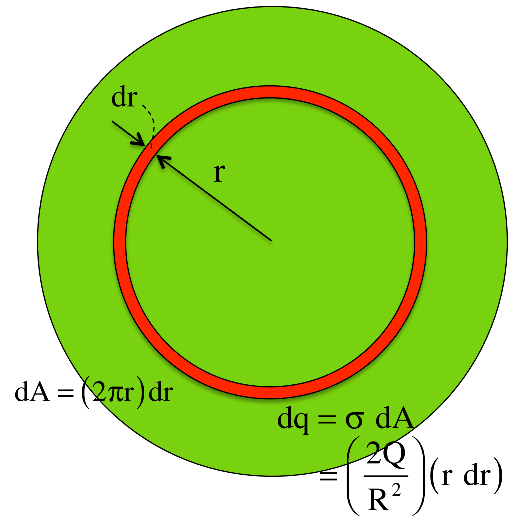
$$\begin{aligned} dA &= (\text{circumference})(\text{thickness}) \\ &= \quad (2\pi r) \qquad\qquad dr \end{aligned}$$

$$\begin{aligned} dq &= \quad \sigma \qquad dA \\ &= \left(\frac{Q_{disk}}{\pi R^2}\right)(2\pi r\ dr) \\ &= \left(\frac{2Q_{disk}}{R^2}\right)(r\ dr) \end{aligned}$$

With all of this information, we are ready to tackle our problem.

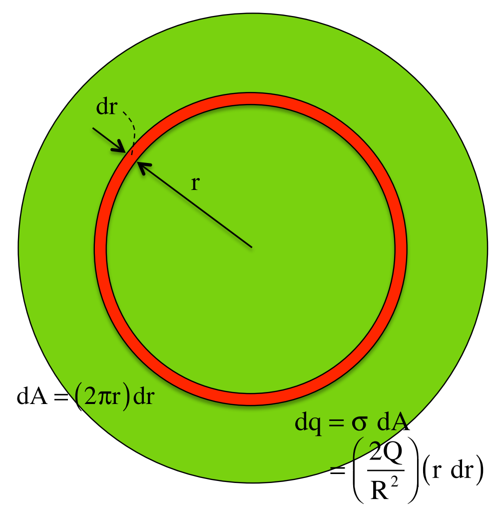
To begin with, we've already determine that the electric field down the central axis of a hoop is along the central axis and has a magnitude of:

$$E = k\frac{Q_{hoop}\ x}{\left(x^2 + R^2\right)^{3/2}}.$$



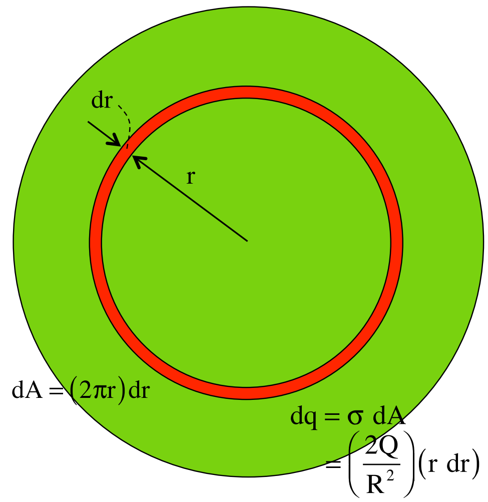
In this problem, E becomes the differential electric field dE due to the swath, Q of the hoop becomes dq of the swath and the radius R of the hoop becomes the radius r of the swath. Making those substitutions, we get:

$$\begin{aligned} dE &= k\frac{dq\ x}{\left(x^2 + r^2\right)^{3/2}} \\ &= k\frac{\left[(\sigma dA)\right]\ x}{\left(x^2 + r^2\right)^{3/2}} \\ &= k\frac{\left[\left(\frac{Q_{disk}}{\pi R^2}\right)(2\pi r\ dr)\right]\ x}{\left(x^2 + r^2\right)^{3/2}} \\ &= k\frac{\left(\frac{2Q_{disk}x}{R^2}\right)(r\ dr)}{\left(x^2 + r^2\right)^{3/2}} \\ &= \left(\frac{2Q_{disk}kx}{R^2}\right)\left(\frac{r}{\left(x^2 + r^2\right)^{3/2}}dr\right) \end{aligned}$$



To determine the total electric field down the axis, we have to sum up all the differential fields due to all the differentially thin hoops from r=0 to r=R. Noting that k, x, Q and R are all constants that can be pulled out of the integral, that operation is shown below.

$$E = \int dE$$
$$= \left(\frac{2kx}{R^2}Q_{disk}\right)\int_{r=0}^{R}\frac{r}{\left(x^2+r^2\right)^{3/2}}dr$$
$$= \left(\frac{2kx}{R^2}Q_{disk}\right)\left(-\frac{1}{\left(x^2+r^2\right)^{1/2}}\right)\Bigg|_{r=0}^{R}$$
$$= -\left(\frac{2kx}{R^2}Q_{disk}\right)\left(\frac{1}{\left(x^2+R^2\right)^{1/2}}-\frac{1}{x}\right)$$

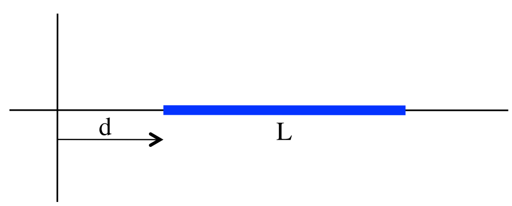


Rearranging the negative signs and putting in the unit vector, this gives us an electric field vector down the central axis of the disk as being:

$$\vec{E} = \left(\frac{2kx}{R^2}Q_{disk}\right)\left(\frac{1}{x}-\frac{1}{\left(x^2+R^2\right)^{1/2}}\right)\left(-\hat{i}\right)$$

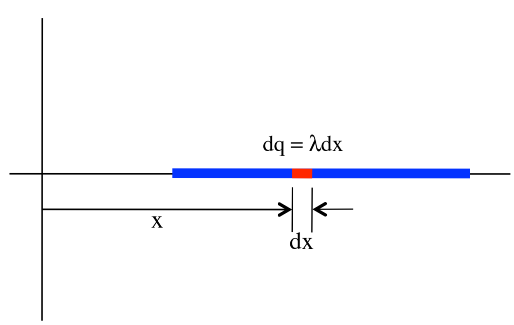
Note: If you use $\sigma = \left(\frac{Q_{disk}}{\pi R^2}\right)$ to derive an expression for Q_{disk}, then substitute that into our expression, you will get the same solution as was acquired in the book.

E.) A thin rod has a net charge Q uniformly distributed throughout it. It is L units long and sits a distance d units from the origin along the x-axis. Derive an expression for the electric field as it exists at (0,0). In fact, set it up for any point in the field.



The (0,0) problem is done in the book on pgs 721, but I didn't like the way it is presented so I'm doing it below in expanded version.

As you have done before, you are going to define an arbitrarily point charge dq residing on a differential length of rod dx located an arbitrary distance x from the coordinate origin (important note: do NOT make the end of the rod your "arbitrary position" . . . it has an actual numeric coordinate—"d+L"—so it ISN'T an arbitrary point!). All of this information MUST BE SHOWN on the layout sketch that you create when doing this problem!!!



NOTE: Having to do with the linear charge density function λ.

If you are not told anything about how the charge is distributed, you will probably do the problem in terms of lambda (λ) and leave it at that. In this problem it was originally stated that there was Q's worth of charge on the rod. That means you can write (and would be expected to write) the charge-per-unit-length ratio as: .

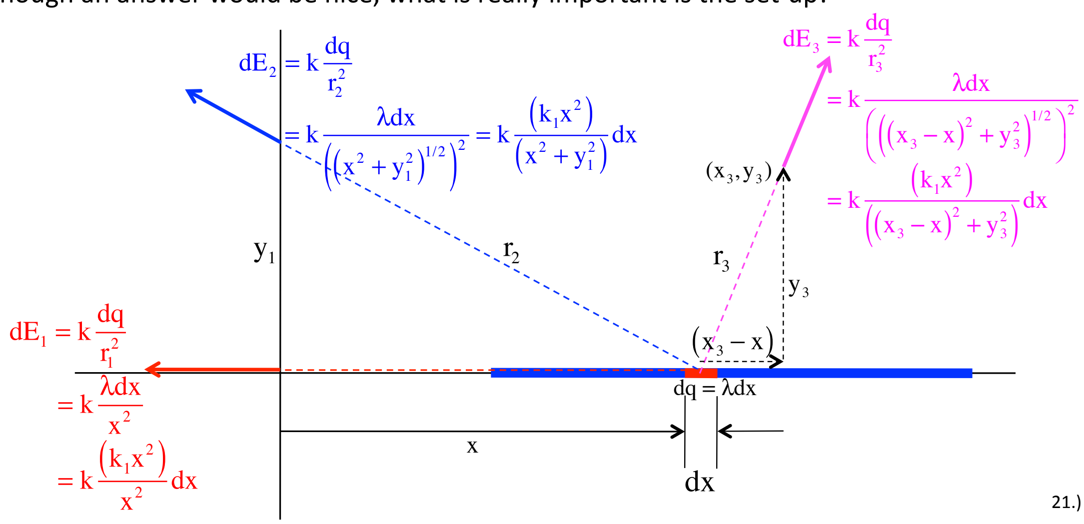
$$\lambda = Q/L$$

In some cases, though, the charge density will be given in terms of x. That is, maybe you are told that $\lambda = k_1x^2$. In that case, dq would still be equal to λdx, but you'd have to take the additional step of substituting in getting: .

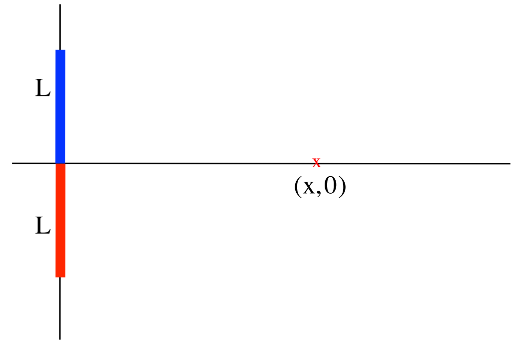
$$\begin{aligned} dq &= \lambda \; dx \\ &= \left(k_1x^2\right)dx \end{aligned}$$

In this problem, just to make it more interesting, I'll finish the problem by changing the lambda to that quoted just above.

Once you have the layout, at the point of interest determine both the DIRECTION and MAGNITUDE of the differential electric field dE produced by the presence of dq. Include any tricky twists that the problem might include (example, substituting in for λ). Break dE into component parts, then utilize symmetry if you can. Once done, integrate to determine the total E-fld due to the presence of all of the differential charges. For our set-up, I've shown in color-coded form the pertinent information for the point (0,0) along with information for a point up the y-axis and for one point "out there" on the plane (I'm leaving the *breaking into components* and *integrating* to your imagination). Though an answer would be nice, what is really important is the set-up!



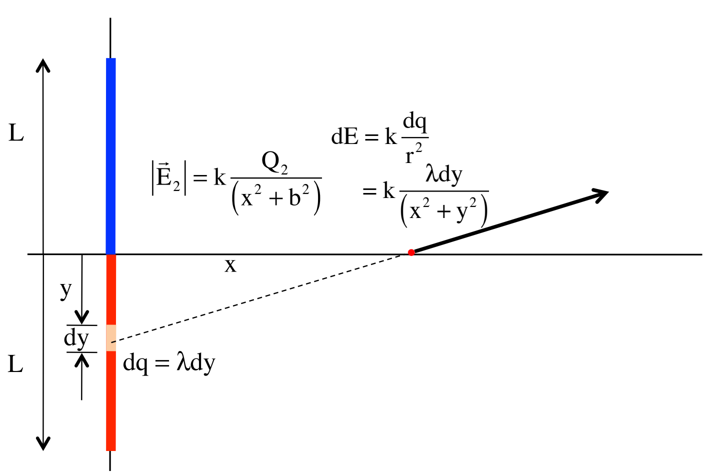
F.) For the point charge configuration shown, derive the expression for the electric field as it exists at (x,0). You may assume that the top half is of length L and has a constant, negative, linear charge distribution $-\lambda$ and the bottom half a constant, positive, linear charge distribution λ.



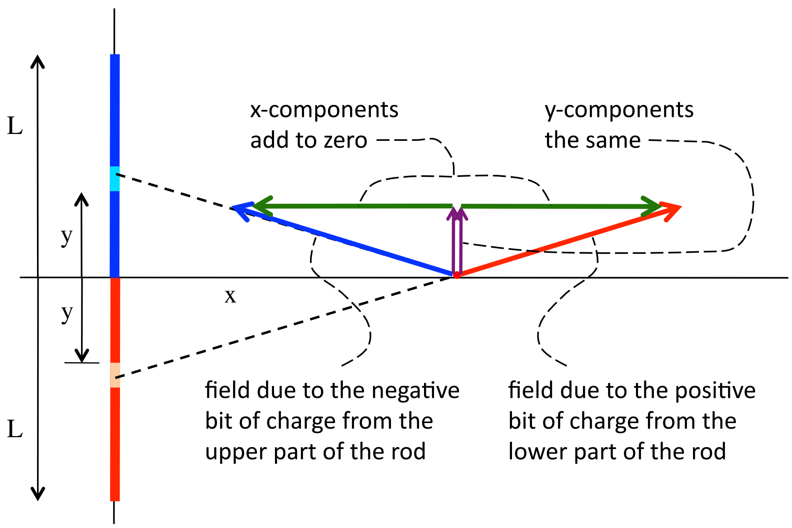
This problem is interesting enough to be examined over the next four pages.

Let's begin by looking at what the positive charge density on the bottom half of the structure is doing at Point (x,0).

Begin by determining the MAGNITUDE and DIRECTION of the differential electric field dE due to an ARBITRARY differential charge dq along a differentially length of rod dy located at an ARBITRARY point "y" units from the origin (see sketch for details of that operation, and remember, the bottom of the rod is NOT arbitrary points).



Notice that there exists a similar section of differential charge on the top half that creates a similar differential electric field, and that due to symmetry the x-components of those two fields will add to zero. In other words, assuming the two rods are the same length and have the same charge-density on them (a sketch of the symmetry is shown below), all we need to do is determine the net vertical component due to the bottom rod, then double it to take into account the top rod.

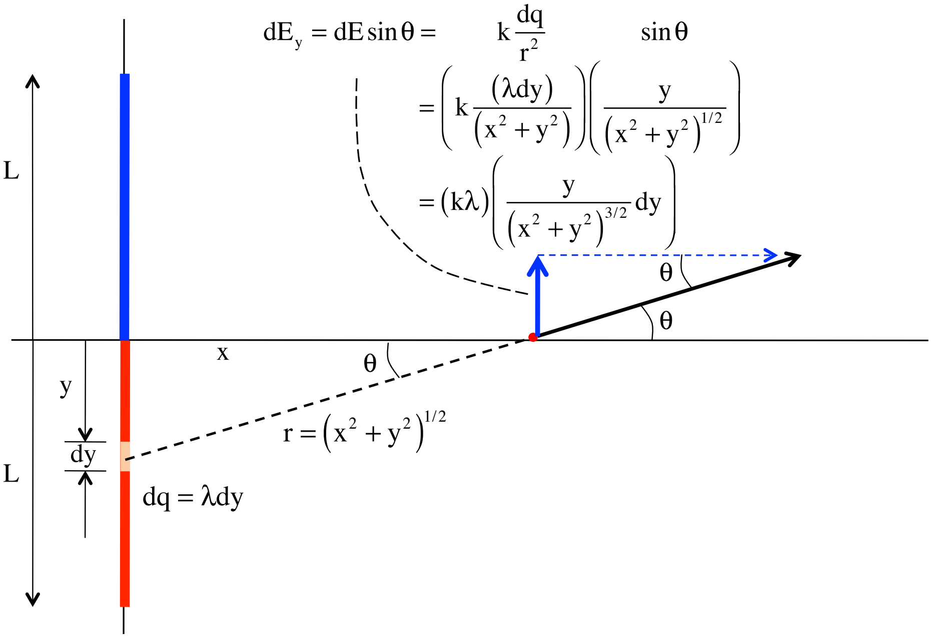


So we are only interested in the y-component. With the angle θ as defined, that need to determine $dE_y = |dE|\sin\theta$. Using the geometry to determine $\sin\theta$ in terms of x's and y's, etc., (that is, $\sin\theta = \frac{y}{\left(x^2+y^2\right)^{1/2}}$), we can write:

$$dE_y = dE\sin\theta = k\frac{dq}{r^2}\sin\theta$$

$$= \left(k\frac{(\lambda dy)}{\left(x^2+y^2\right)}\right)\left(\frac{y}{\left(x^2+y^2\right)^{1/2}}\right)$$

$$= (k\lambda)\left(\frac{y}{\left(x^2+y^2\right)^{3/2}}dy\right)$$



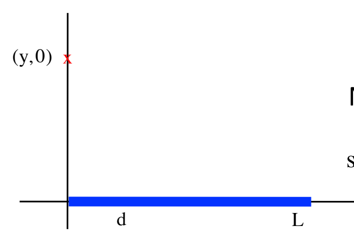
This is the function we want to integrate from y=0 to y=L. Doing so yields:

$$
\begin{aligned}
dE\sin\theta &= \quad k\frac{dq}{r^2} \qquad\qquad \sin\theta \\
&= \left(k\frac{(\lambda dy)}{(x^2+y^2)}\right)\left(\frac{y}{(x^2+y^2)^{1/2}}\right) \\
&= (k\lambda)\left(\frac{y}{(x^2+y^2)^{3/2}}dy\right)
\end{aligned}
$$

Noting that we are multiplying by 2 to take care of the top rod, the net electric field in the y-direction will be:

$$
\begin{aligned}
E = E_y &= 2(k\lambda)\int_{y=0}^{L}\left(\frac{y}{(x^2+y^2)^{3/2}}\right)dy \\
&= 2(k\lambda)\left(-\frac{1}{(x^2+y^2)^{1/2}}\right)\Bigg|_{y=0}^{L} \\
&= -2(k\lambda)\left(\frac{1}{(x^2+L^2)^{1/2}}-\frac{1}{x}\right)
\end{aligned}
$$

G.) A thin rod has a charge density of along it's length. Its total length is L units long and it sits as shown. Derive an $\lambda = kx$ expression for the electric field as it exists at $(y_1, 0)$.

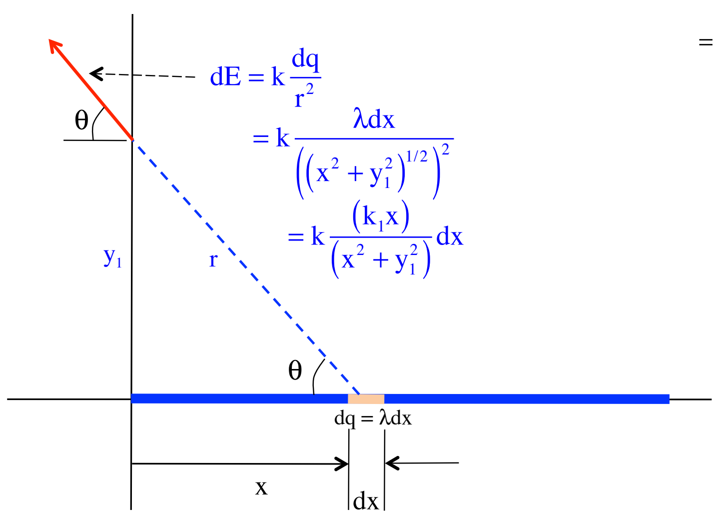


Note that:

$$
\begin{aligned}
\sin\theta &= \frac{y_1}{r} \\
&= \frac{y_1}{(x^2+y_1^2)^{1/2}}
\end{aligned}
$$

$$
\begin{aligned}
\cos\theta &= \frac{x}{r} \\
&= \frac{x}{(x^2+y_1^2)^{1/2}}
\end{aligned}
$$

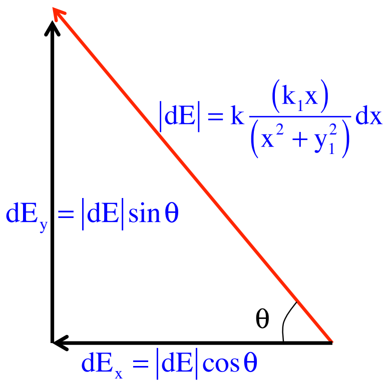
If you understood Problem E, this shouldn't be too hard to set up. The solution follows. (Note that I've defined an angle θ for future use.)



$$
\begin{aligned}
dE &= k\frac{dq}{r^2} \\
&= k\frac{\lambda dx}{\left((x^2+y_1^2)^{1/2}\right)^2} \\
&= k\frac{(k_1 x)}{(x^2+y_1^2)}dx
\end{aligned}
$$

We need the x and y-components. Using the sine and cosine tricks, we get:

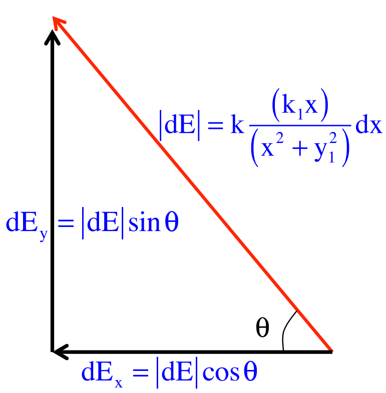
y-direction



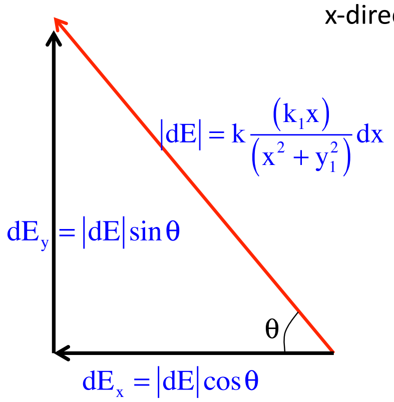
$$
\begin{aligned}
E_y &= \int dE_y \\
&= \int \quad |dE| \quad\quad \sin\theta \\
&= \int \left(k\frac{(k_1 x)}{(x^2 + y_1^2)}dx\right)\left(\frac{y_1}{(x^2 + y_1^2)^{1/2}}\right) \\
&= kk_1 y_1 \int_{x=0}^{L} \left(\frac{x}{(x^2 + y_1^2)^{3/2}}\right)dx \\
&= kk_1 y_1 \left(-\frac{1}{(x^2 + y_1^2)^{1/2}}\right)\Bigg|_{x=0}^{L} \\
&= -kk_1 y_1 \left(\frac{1}{(L^2 + y_1^2)^{1/2}} - \frac{1}{y_1}\right)
\end{aligned}
$$

We need the x and y-components. Using the sine and cosine tricks, we get:

x-direction



I didn't know off-hand the solution to this integral, so I used a table of integrals. If you run into something like this on a test, you can leave it in integral form (i.e., go through line 4 and including the limits).

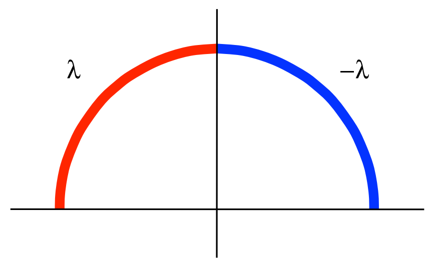


x-direction

$$E_x = \int dE_x$$
$$= \int \quad |dE| \quad \cos\theta$$
$$= \int\left(k\frac{(k_1 x)}{(x^2+y_1^2)}dx\right)\left(\frac{x}{(x^2+y_1^2)^{1/2}}\right)$$
$$= kk_1y_1\int_{x=0}^{L}\left(\frac{x^2}{(x^2+y_1^2)^{3/2}}\right)dx$$
$$= kk_1y_1\left(-\frac{x}{(x^2+y_1^2)^{1/2}} + \log\left(x+(x^2+y_1^2)^{1/2}\right)\right)\Big|_{x=0}^{L}$$
$$= -kk_1y_1\left[\left(\frac{L}{(L^2+y_1^2)^{1/2}} - \log\left(L+(L^2+y_1^2)^{1/2}\right)\right) - (-\log y_1)\right]$$
$$= -kk_1y_1\left[\left(\frac{L}{(L^2+y_1^2)^{1/2}} - \log\left(L+(L^2+y_1^2)^{1/2}\right)\right) + \log y_1\right]$$
$$= -kk_1y_1\left[\frac{L}{(L^2+y_1^2)^{1/2}} + \log\frac{y_1}{\left(L+(L^2+y_1^2)^{1/2}\right)}\right]$$

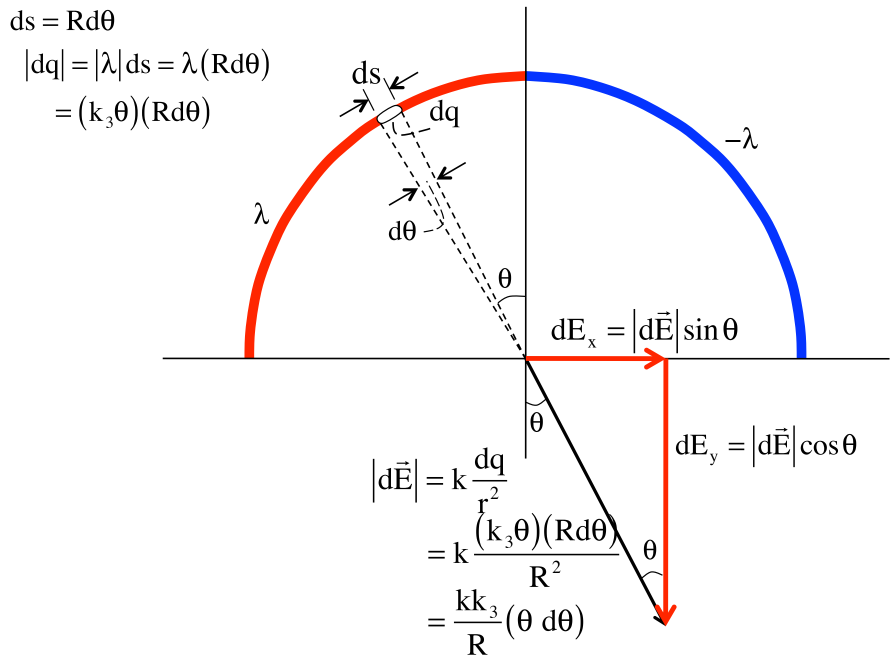
I didn't know off-hand the solution to this integral, so I used a table of integrals. If you run into something like this on a test, you can leave it in integral form (i.e., go through line 4 and including the limits).

H.) A quarter-circle rod in red has a charge density of $\lambda = k_3\theta$ along it's length, where θ is an angular magnitude measured from the vertical in radians (see sketch). A second quarter-circle rod in blue has a similar charge density that differs only in that it is positive charge along it's length. If the radius of the two rods is R, derive an expression for the electric field as it exists at (0,0).



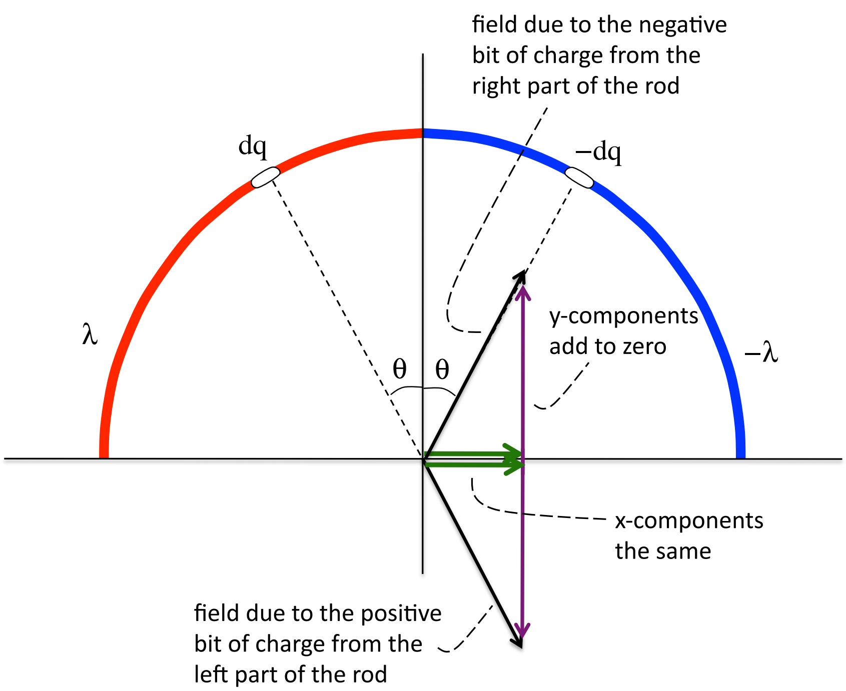
As usual, pick a differential amount of charge dq, relate it to lambda via ds, determine the differential electric field dE at the origin, break dE into its component parts, exploit symmetry and do the final integral. Easy!!! All of that is shown on the last few pages.

We begin by defining ds, relating it to dq, then determine the direction and magnitude of dE at the origin. All of that is shown on the sketch.



32.)

Exploiting symmetry, we can see that the y-components of the net field will go to zero (see sketch below), so all we have to deal with is the x-component.



33.)

So we are interested in the x-component of the field. The integral we have to deal with looks like:

$$\begin{aligned} E &= 2\int dE_x = 2\int \left|d\vec{E}\right| \sin\theta \\ &= 2\int \left(\frac{kk_3}{R} (\theta \, d\theta) \right) \sin\theta \\ &= \left(\frac{2kk_3}{R} \right) \int_{\theta=0}^{\pi/2} (\theta \ \sin\theta) d\theta \end{aligned}$$

How do you do this integral? Physicists tend to be rather casual about the math because math is a means to an end, not the be-all, end-all (as is the case with mathematicians). As such, if I'd done this problem on a test and didn't know how to evaluate the integral, I'd either leave it as is or look the integral up in a book of integrals (assuming that was a kosher move on a test). Unfortunately, I left my book of integrals at home, so I did the next best thing. I asked both Mr. Fay and Mr. Strom for the solution. Both obliged by deriving it using "integration by part," a technique I learned forty years ago and promptly forgot. In any case, the solution to this problem is shown on the next page. (Please note, this is not an endorsement for blowing off your knowing your Calculus! It's more me being honest than anything else.)

So we are interested in the x-component of the field. The integral we have to deal with looks like:

$$\begin{aligned} E &= \left(\frac{2kk_3}{R} \right) \int_{\theta=0}^{\pi/2} (\theta \ \sin\theta) d\theta \\ &= \left(\frac{2kk_3}{R} \right) \left[-\theta \ \cos\theta + \sin\theta \right] \Big|_{\theta=0}^{\pi/2 \text{ radians}} \\ &= \left(\frac{2kk_3}{R} \right) \left[\left(-\left(\frac{\pi}{2}\right) \ \cos\left(\frac{\pi}{2}\right) + \sin\left(\frac{\pi}{2}\right) \right) - \left(-(0) \ \cos 0 + \sin 0 \right) \right] \\ &= \left(\frac{2kk_3}{R} \right) \left[(-0+1) - (-0+0) \right] \\ &= \left(\frac{2kk_3}{R} \right) \end{aligned}$$

As a vector, the net electric field at the origin would be a constant:

$$\hat{E} = \left(\frac{2kk_3}{R} \right) \hat{i}$$